# A NEW $K-\varepsilon$ TURBULENCE PARAMETERIZATION FOR MESOSCALE METEOROLOGICAL MODELS

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## Abstract

A new one-dimensional 1.5-order planetary boundary layer (PBL) scheme, based on the $K-\varepsilon$ turbulence closure applied to the Reynolds-averaged-Navier-Stokes (RANS) equations, is developed and implemented within the Weather Research and Forecasting (WRF) model. The new scheme includes an analytic solution of the coupled equations of the turbulent kinetic energy and of the dissipation rate. Different versions of the PBL scheme are proposed, with increasing levels of complexity, including a model for the calculation of the Prandtl number, a correction to the dissipation rate equation, and a prognostic equation for the temperature variance. Five different idealized cases are investigated: four of them explore convective conditions, and they differ in initial thermal stratification and terrain complexity, while one simulates the very stable boundary layer case known as GABLS. For each case study, an ensemble of different Large Eddy Simulations (LES), has been taken as reference for the comparison with the novel PBL schemes and other state-of-the-art 1- and 1.5-order turbulence closures. Results show that the new PBL $K-\varepsilon$ scheme brings improvements in all the cases tested in this study. Specifically, the largest enhancements are brought by the turbulence closure including a prognostic equation for the temperature variance. Moreover, the largest benefits are obtained for the idealized cases simulating a typical thermal circulation within a two-dimensional valley. This suggests that the use of prognostic equations for the dissipation rate and temperature variance, which take into account their transport and history, is particularly important with increasing complexity of PBL dynamics.

## The Model

Zonal wind speed:

$$\frac{\partial U}{\partial t} = -\frac{\partial \overline{uw}}{\partial z} \tag{1a}$$

Meridional wind speed:

$$\frac{\partial V}{\partial t} = -\frac{\partial \overline{vw}}{\partial z} \tag{1b}$$

Potential temperature:

$$\frac{\partial \Theta}{\partial t} = -\frac{\partial \overline{w\theta}}{\partial z} \tag{1c}$$

TKE:

$$\frac{\partial K}{\partial t} = \underbrace{-\frac{\partial \overline{wk}}{\partial z}}_{\text{v. diffusion}} \underbrace{-\overline{uw}\frac{\partial U}{\partial z} - \overline{vw}\frac{\partial V}{\partial z}}_{\text{shear prod.}} + \underbrace{\frac{g}{\Theta_0}\overline{w\theta}}_{\text{buoy. prod./destr.}} - \underbrace{\varepsilon}_{\text{dissipation}} \tag{1d}$$

Dissipation Rate:

$$\frac{\partial \varepsilon}{\partial t} = \underbrace{-\frac{1}{\sigma_\varepsilon}\frac{\partial \overline{\varepsilon w}}{\partial z}}_{\text{v. diffusion}} - \underbrace{\left[c_1\left(\overline{uw}\frac{\partial U}{\partial z} + \overline{vw}\frac{\partial V}{\partial z}\right) - c_3\frac{g}{\Theta_0}\overline{w\theta}\right]}_{\text{shear+buoy. prod./destr.}} \underbrace{\frac{\varepsilon}{K}}_{1/\tau} - \underbrace{c_2\frac{\varepsilon^2}{K}}_{\text{dissipation}} \tag{1e}$$

Where:

$$\overline{uw} = -\nu_M\frac{\partial U}{\partial z} \qquad \overline{vw} = -\nu_M\frac{\partial V}{\partial z} \qquad \overline{\theta w} = -\nu_H\frac{\partial \Theta}{\partial z} \tag{2}$$

$$\nu_M = c_\mu\frac{K^2}{\varepsilon} \qquad \nu_H = \nu_M/Pr \qquad Pr = 1 + (Pr_0 - 1)\exp\left[\frac{-3\,(z-0.1h)^2}{h^2}\right] \tag{3}$$

1) $K-\varepsilon-\gamma$:

$$\overline{w\theta} = -\nu_H\left(\frac{\partial \Theta}{\partial z} - \gamma\right) \qquad \gamma = C\frac{\overline{w\theta}_s}{w_* h} \tag{4}$$

2) $K-\varepsilon-\theta^2$:

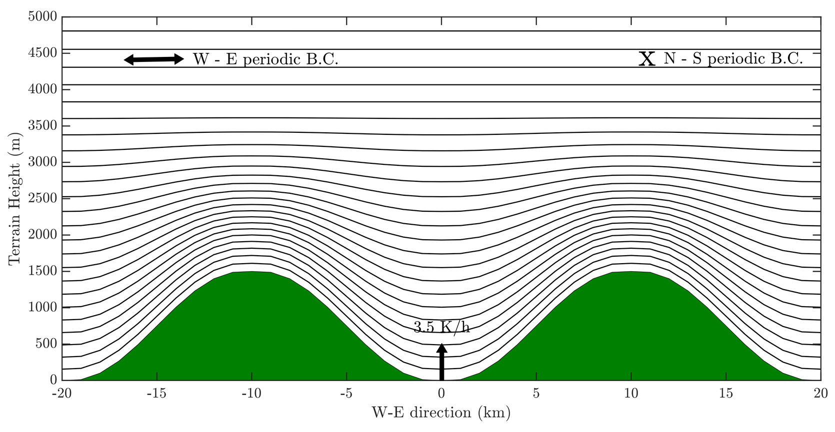
$$\overline{w\theta} = -\nu_H\frac{\partial \Theta}{\partial z} + \Phi_{cg} \qquad \Phi_{cg} = c_\mu\frac{g}{\Theta_0}\frac{K K_\theta}{\varepsilon} \tag{5}$$

with prognostic equation of temperature variance:

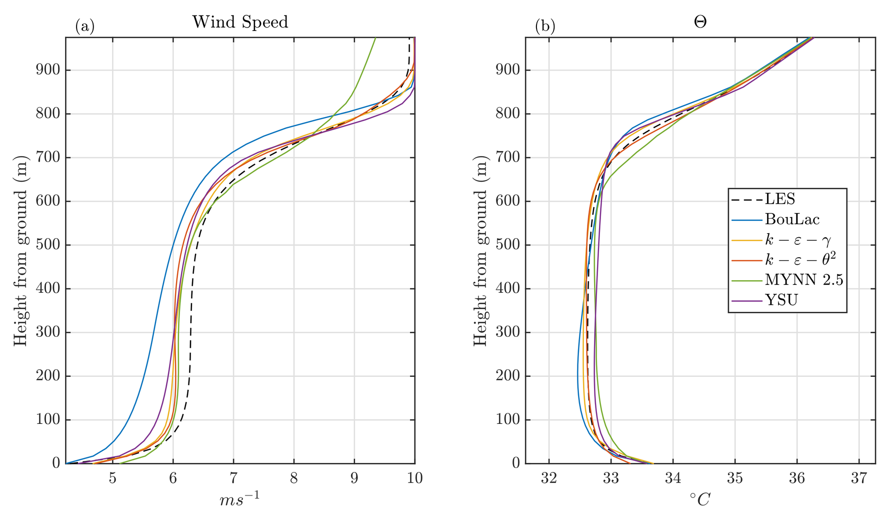
$$\frac{\partial K_\theta}{\partial t} = -\frac{\partial \overline{wK_\theta}}{\partial z} - \overline{w\theta}\frac{\partial \Theta}{\partial z} - \varepsilon_\theta \tag{6}$$

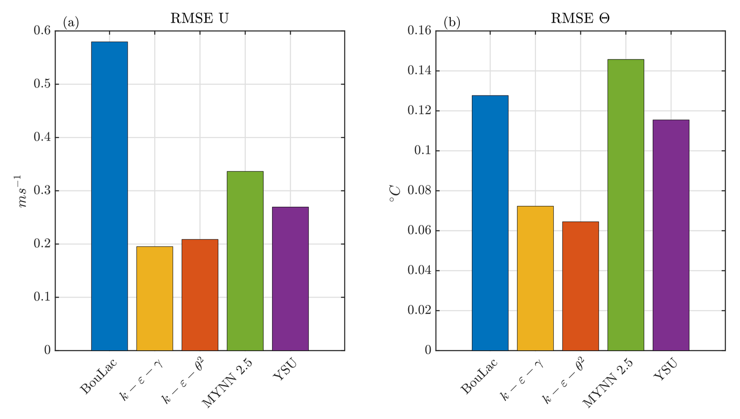
## Idealized cases study

| | PBL type | $dT_S/dt$ (K h$^{-1}$) | $d\Theta_0/dz$ (K km$^{-1}$) | Terrain | $U_g$ (m s$^{-1}$) | $V_g$ (m s$^{-1}$) | Domain Size (km × km × km) |
|---|---|---|---|---|---|---|---|
| CBL_F_3 | Convective | 3.5 | 3.3 | Flat | 0 | 10 | 10 × 10 × 3 |
| CBL_F_10 | Convective | 3.5 | 10 | Flat | 0 | 10 | 10 × 10 × 3 |
| CBL_V_NOW | Convective | 3.5 | 3.3 | Valley | 0 | 0 | 40 × 10 × 5 |
| CBL_V_W | Convective | 3.5 | 3.3 | Valley | 0 | 10 | 40 × 10 × 5 |
| GABLS | Stable | -0.25 | 10 | Flat | 8 | 0 | 10 × 10 × 1 |

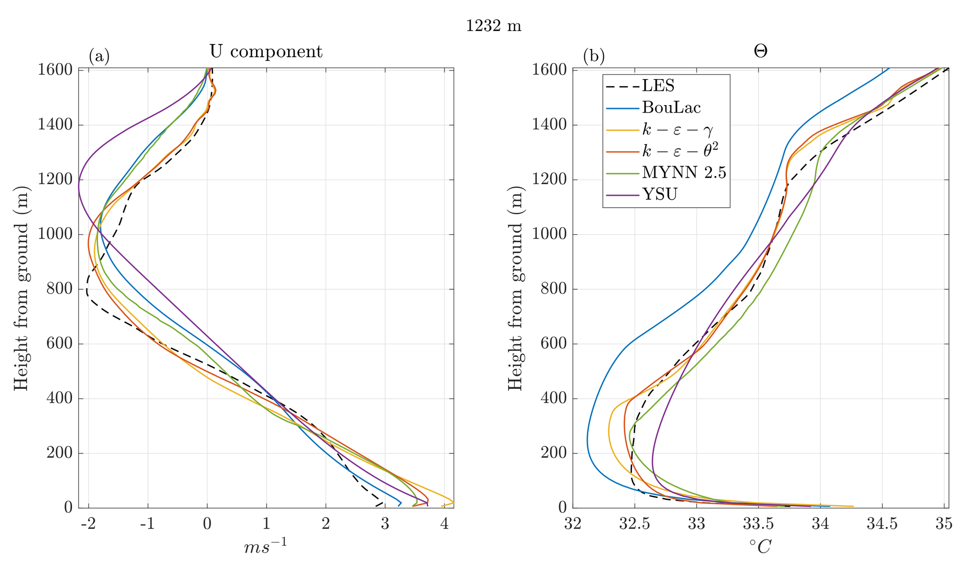


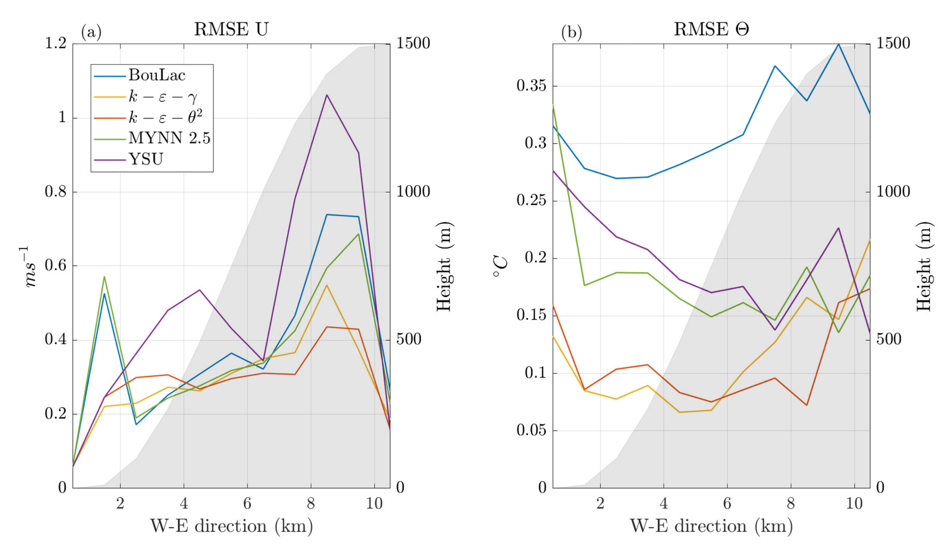

## Results - Flat convective PBL



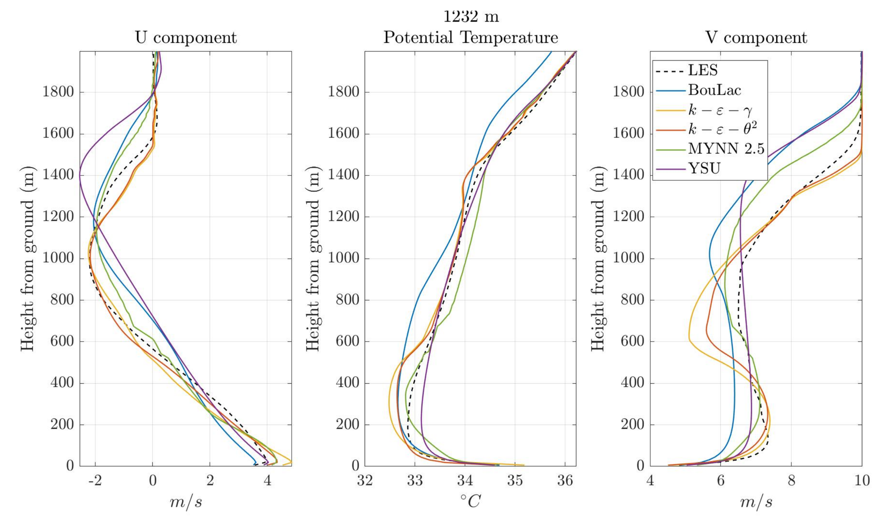



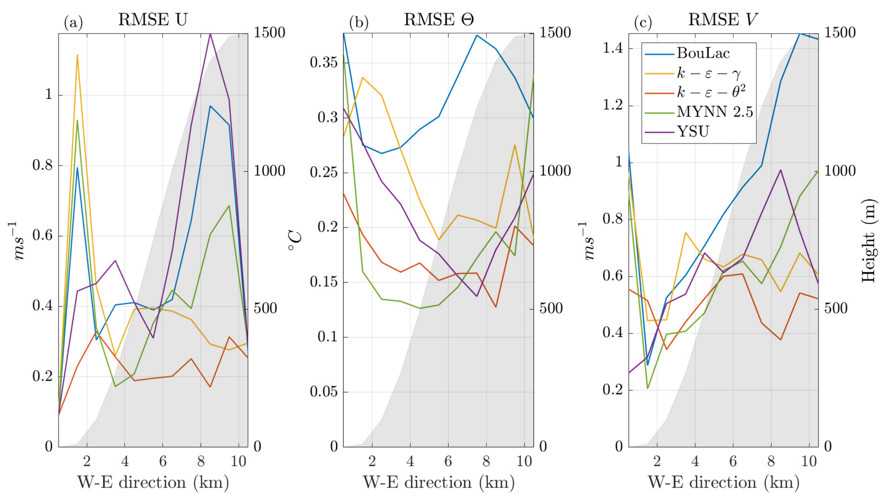

## Results - Valley convective PBL NW



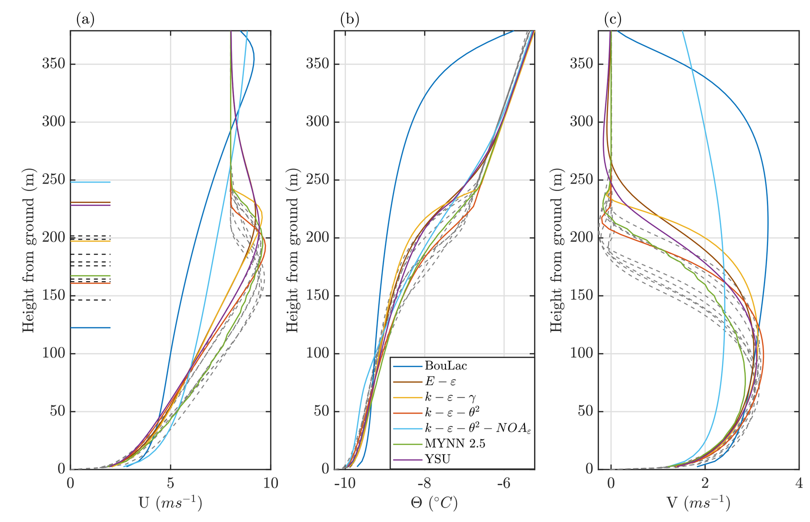



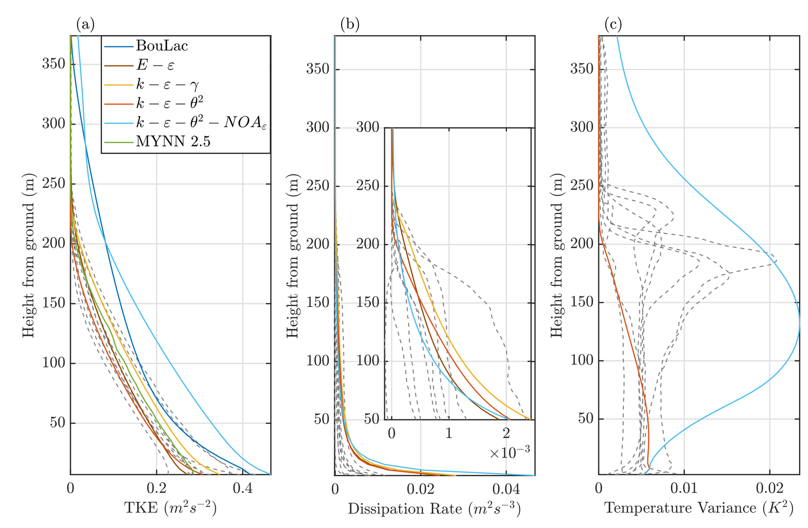

## Results - Valley convective PBL WW

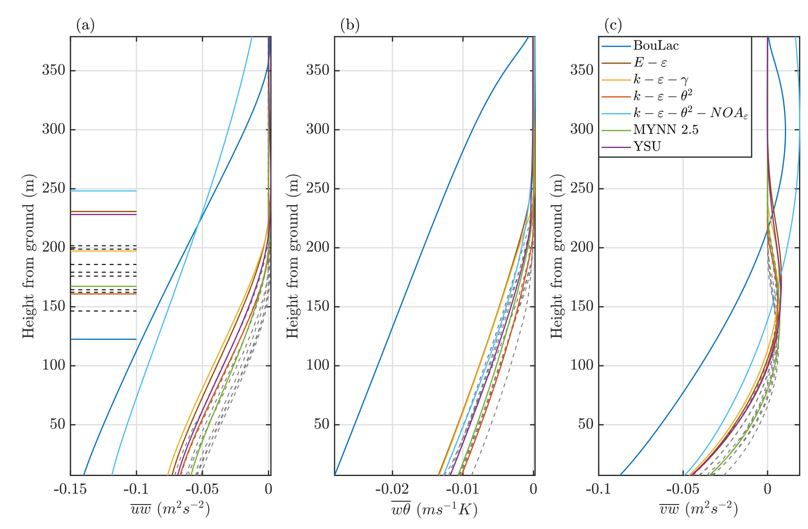



## Results - Flat stable PBL (GABLS)







## Conclusions

Results show that in general the novel $K-\varepsilon$ scheme outperforms the other parameterizations, in all the cases considered in this work. In particular, the largest improvements take place in connection with inversion layers, where the gradients of the mean variables are stronger. The improvement of model performance increases with the increasing complexity of the atmospheric conditions, as for the valley cases, where the enhancements are substantial.
The comparison between the various $K-\varepsilon$ closures, differing in the calculation of the counter-gradient term for the turbulent heat flux, underlines the importance of adopting a prognostic equation for the temperature variance $\overline{\theta^2}$. Improvements due to the prognostic equation of temperature variance are evident both in the valley and GABLS cases. Finally, this work proves that the new scheme discussed here can improve the reproduction of the atmospheric motion in several conditions, going beyond the definition of a diagnostic turbulent length scale commonly adopted in state-of-the-art PBL closures.

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