# SOLSTICIAL HADLEY CELL ASCENDING EDGE THEORY FROM SUPERCRITICALITY

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## The Hadley Cell

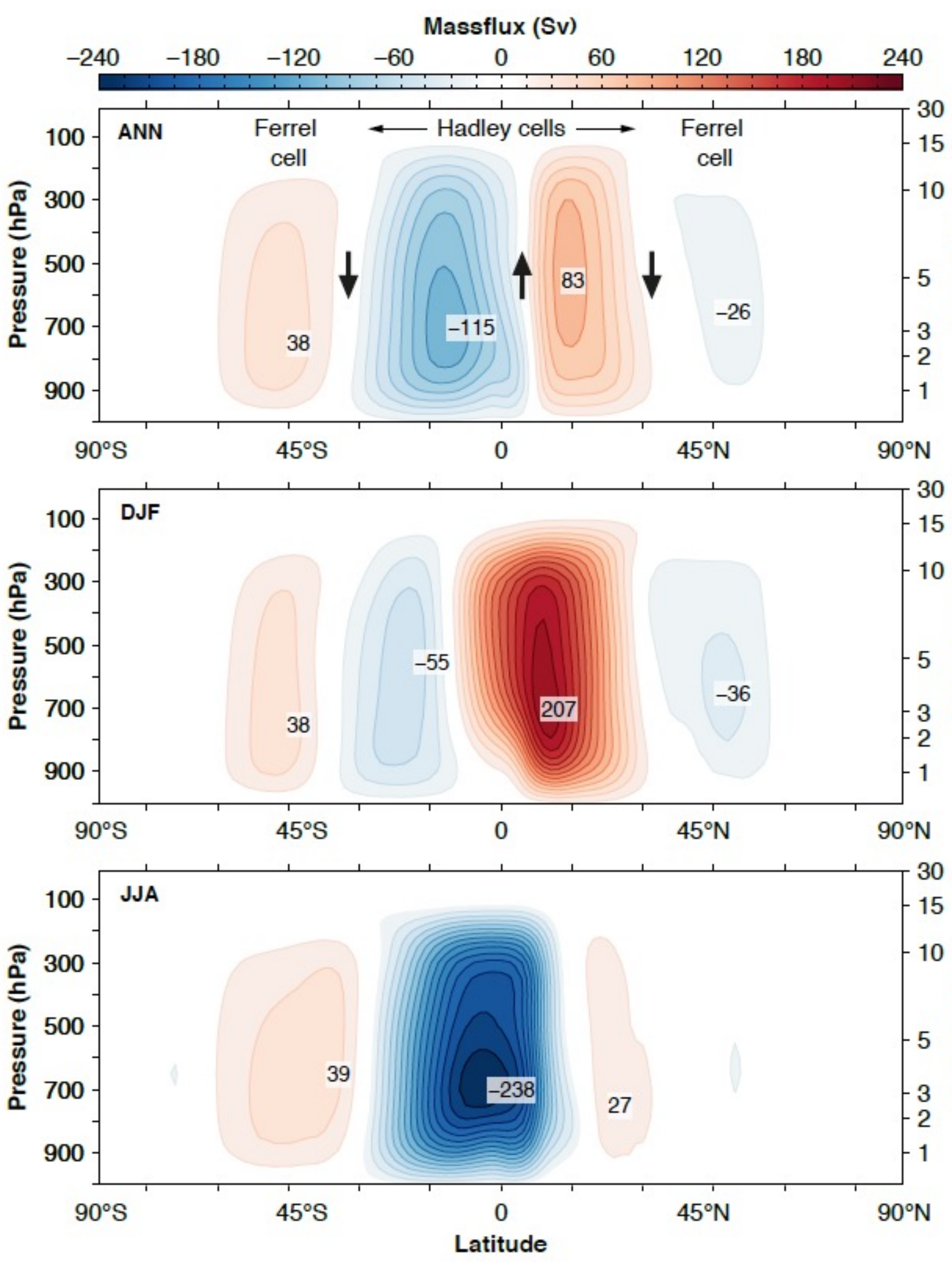


**Figure 1:** Streamfunction of meridional mass circulation in the annual mean (top), DJF mean (middle), and JJA mean (bottom) from ERA-Interim.

Why does the shared, ascending edge of Earth's Hadley cells sits at ~15° in the summer hemisphere? Several theories exist of direct or indirect relevance to this fundamental property of the general circulation of the atmosphere, but each is limited in one or more substantive ways. Here, we pursue a predictive theory for the edge of the ascending branch of the cross-equatorial Hadley cell based on the extent of supercritical radiative forcing. A supercritical latitude is one at which, supposing no large-scale overturning circulation existed, the resulting state of latitude-by-latitude radiative–convective equilibrium (RCE) would possess impermissible distributions of angular momentum and absolute vorticity. A large-scale overturning circulation must therefore span at minimum all supercritical latitudes.

## The RCE state

We use the **climlab** single-column model (Rose 2018) to simulate solsticial latitude-by-latitude RCE, forced with insolation corresponding to present-day, boreal summer solstice, with the chosen latitudes in 1° increments spanning from the equator to the pole in the summer hemisphere and from the equator to 55° in the winter hemisphere. Time-averaged fields from the single-column simulations are concatenated together in latitude to yield latitude–pressure distributions of each field. Figure 2 shows the resulting temperature field T. Once the temperature distribution is known, we compute:

- The zonal wind u, assuming thermal wind balance;
- Angular momentum using:

$$M = a\cos\varphi(\Omega a\cos\varphi + u)$$

- The absolute vorticity, defined as:

$$\eta = \frac{-1}{a^2\cos\varphi}\frac{\partial M}{\partial\varphi} = f + \zeta$$

Figure 3 shows the meridional profile of absolute vorticity of the RCE state. It is negative up to ~15°N, which defines the supercritical latitudinal range and constitutes the poleward extent of the resulting Hadley cell.

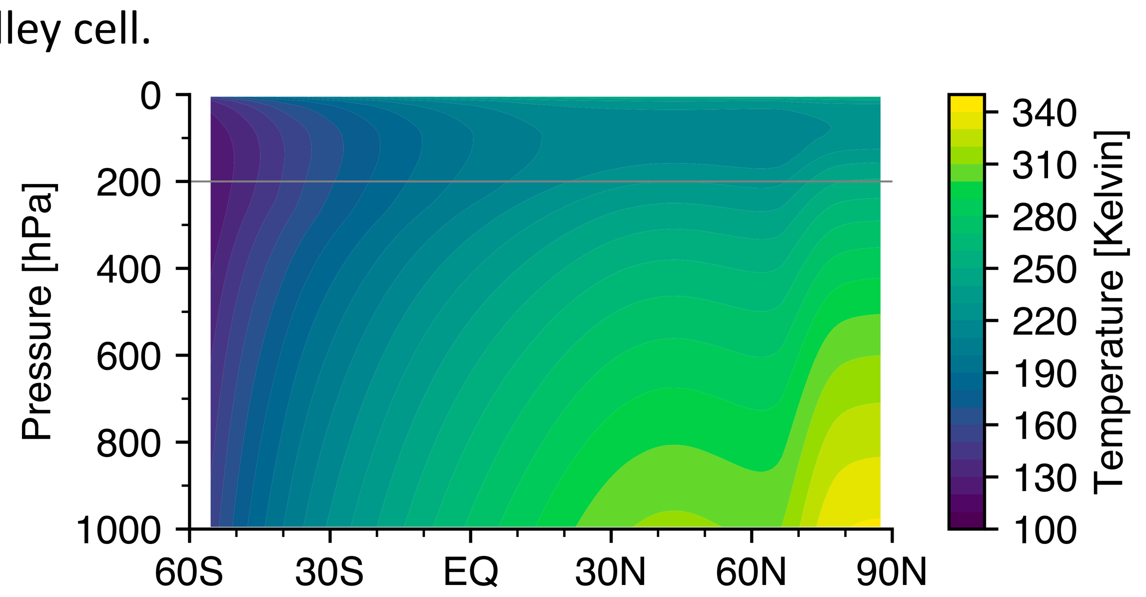


**Figure 2:** Temperature as a function of latitude and pressure from the solsticial RCE simulation. The gray line indicates the 200 hPa level at which the temperature is used to compute the thermal wind.

## Analytical solution

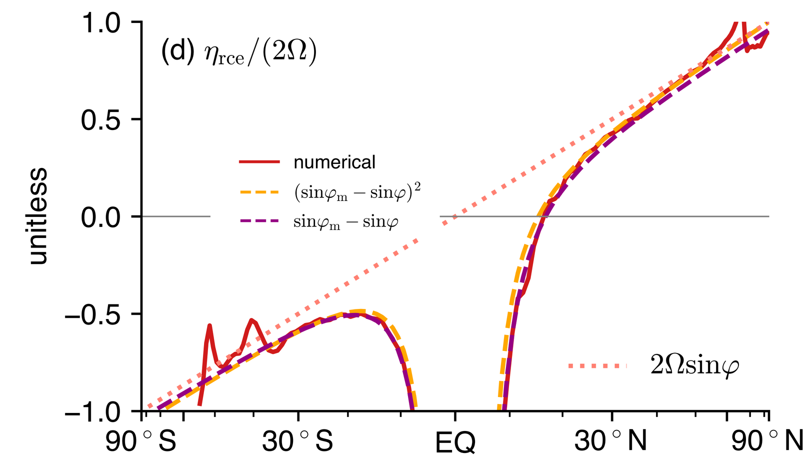


**Figure 3:** Upper-tropospheric absolute vorticity of the RCE state.

We approximate the RCE temperature with the following analytical profile (Lindzen and Hou, 1988)

$$\frac{\hat{\theta}_{rce}}{\theta_0} = 1 + \frac{\Delta_h}{3}[1 - 3(\sin\varphi_m - \sin\varphi)^2]$$

and compute the absolute vorticity from the zonal winds in thermal wind balance with this thermal forcing.

It can be shown that in the small angle approximation, the absolute vorticity changes sign at latitude

$$\varphi_c = \left(\frac{\mathrm{Ro}_{th}}{2}\right)^{1/3}$$

with thermal Rossby number $\mathrm{Ro}_{th} \equiv \frac{gH}{\Omega^2 a^2}\Delta_h \sin\varphi_m$

This latitude represents the extent of the supercritical forcing and hence the minimum extent of the resulting Hadley cell.

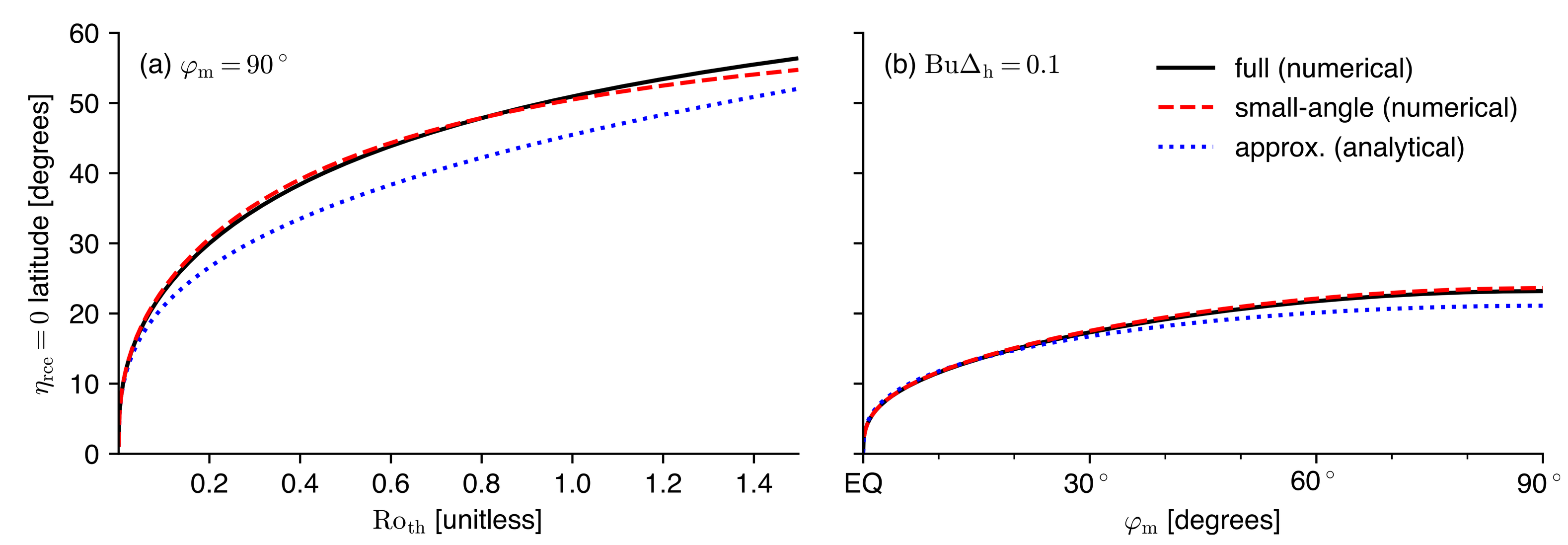


**Figure 4:** Supercritical forcing extent as a function of thermal Rossby number and latitude of maximal forcing.

## Idealized GCM simulations

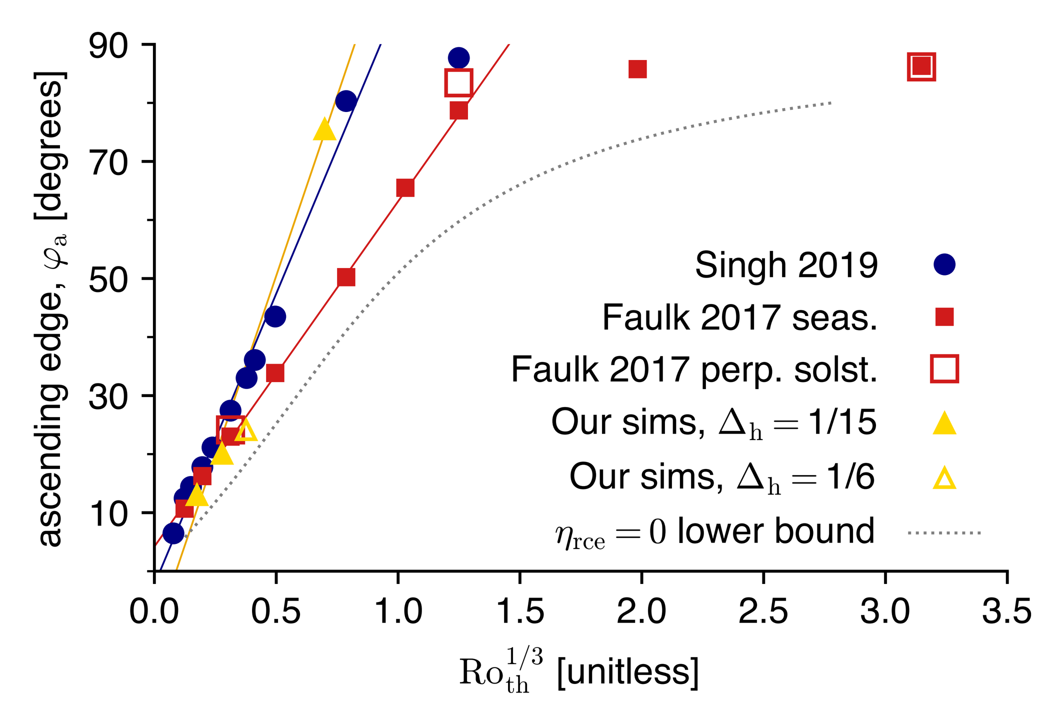


**Figure 5:** Cross-equatorial Hadley cell edge in the summer hemisphere in idealized moist and dry GCM simulations.

We examine the ascending edge latitude in simulations performed in two variants of an idealized, moist GCM and an idealized dry GCM, across each of which planetary rotation rate is varied. Under solsticial conditions, in each model the cross-equatorial Hadley cell expands as the rotation rate decreases, and for diagnosed values of $\mathrm{Ro}_{th}$ up to order unity, this expansion follows the $\mathrm{Ro}_{th}^{1/3}$ scaling predicted by our approximate solution. Simulations with very slow rotation rates and thus large $\mathrm{Ro}_{th}$ values deviate from the scaling, but in a way that qualitatively resembles the more general solution (solved numerically). While our theory is accurate only somewhat qualitatively, relies on some empiricism and provides a lower bound rather than a precise prediction, it is an important step forward in developing a theory for a fundamental aspect of the Hadley cell, such as the position of its ascending branch.

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